

Teaching Mathematical Reasoning in Science, Engineering, and Technology

Pradip Peter Dey, Ronald P. Uhlig, Mohammad N. Amin,
Arun Datta, Gordon W. Romney, Thomas M. Gatton,
Mudasser F. Wyne, and Albert P. Cruz

Abstract

Teaching mathematical reasoning is a challenge for most result-oriented teachers. In general, many strategies can be employed, including problem-based learning, technology-based learning, game-based learning, community-based learning, work-based learning, inquiry-based learning, project-based learning, team-based learning, web-based learning and participatory learning. However, none of these strategies may address the central problem of mistakes made with inappropriate application of intuition in mathematical problem solving. This paper emphasizes an agile method of teaching rapid reconciliation of intuition and controlled mathematical reasoning to engineering students in order to overcome inappropriate use of the intuitive mode of cognitive function. This emphasis is based on an extensive review of existing research and an emerging understanding of interactions between intuition and the controlled mode of cognitive function.

Key Words

Intuition, agile teaching, access, metacognitive strategies, algebraic thinking

Introduction

Daniel Kahneman, in his 2002 Nobel Prize lecture, distinguished “two generic modes of cognitive function: an intuitive mode in which judgments and decisions are made automatically and rapidly, and a controlled mode, which is deliberate and slower” (Kahneman, 2002). Kahneman and other researchers have collected experimental results showing that judgments and decisions made in intuitive mode are frequently erroneous (Alter, Openheimer, Epley, & Eyre, 2007; Evans, 2003; Kahneman & Frederick 2002). In his Nobel Prize lecture, Kahneman mentioned several experiments including the following:

A bat and a ball cost \$1.10 in total. The bat costs \$1 more than the ball. How much does the ball cost?

What is remarkable is that almost everyone has an initial tendency to answer “10 cents” because “the sum \$1.10 separates naturally into \$1 and 10 cents” (Kahneman, 2002). It was found that

50% of Princeton students and 56% of students at the University of Michigan gave the wrong answer. The correct answer is "5 cents" which is reached through the controlled mode (the bat costs \$1.05, which is a dollar more than the 5-cent ball). However, intuitive thoughts come to mind spontaneously like percepts, whereas controlled thoughts do not come effortlessly. Those who gave the correct answer after overcoming their initial tendency have likely utilized the controlled mode of cognitive function in a deliberate way.

Intuitive thoughts are not useless. For example, intuitive judgments about love, affection, and family matters are usually good. However, in engineering, science and technology, students should be able to use mathematical reasoning correctly. "Recent test results show that U.S. 10th-graders ranked just 17th in science among peers from 30 nations, while in math they placed in the bottom five" (Wallis, 2008). Within the United States there are a number of other variations including urban and suburban. Many teaching strategies that have been tried show important improvements in student learning in different settings (Borman, 2005). However, significant nationwide improvements have not been achieved despite these isolated demonstrations of success.

It is not credible that culture, nationality, race, ethnicity, or religion would have anything to do with mathematical reasoning. Religious, ethnic, and socioeconomic groups may show intuitive differences but must agree with mathematical reasoning such as

$$3x = 18$$

therefore $x = 6$

We would all agree with this reasoning despite any differences in religion, culture, or political philosophy. Analyses of the serious problems we face today need to be carefully formulated mathematically. The trade deficit, credit crunch, mortgage meltdown, and high cost of oil imports are examples of problems that need to be analyzed mathematically so that remedies can be worked out without any biases. People's immediate answers to these problems come from intuition. However, use of intuition to solve such problems may give misleading answers. Making correct decisions based on mathematical reasoning should be an ideal goal (Mingus & Grassl, 1998). How do we ensure that we arrive at the correct answers for such problems? Two important considerations are required to address this question: accessibility of thoughts and metacognitive strategies.

The first major consideration is what Kahneman (2002) calls "the relative accessibility" of different thoughts. If someone does not know how to solve linear equations, then problem solving with linear

equations is inaccessible to that person. A more interesting case is when one knows how to solve linear equations but does not have sufficient practice in problem solving with linear equations; intuition may often play a dominant role in the thoughts of such a person. Accessibility is the relative ease with which particular mental contents come to mind (Higgins, 1996). Some research indicates that intuitive errors are less likely to be corrected when people are under cognitive load or respond quickly (Bless & Schwarz, 1999; Chaiken, 1980; Petty & Cacioppo, 1986). Other research shows that intuitive errors are more likely to be corrected when people are accountable for their judgments (Tetlock & Lerner, 1999). A major goal of engineering instruction is to strengthen the mathematical foundations of engineering students. Algebraic thinking should be promoted in engineering problem-solving environments (Kriegler, 2008). This paper describes ongoing efforts to increase accessibility of mathematical reasoning by applying a variety of teaching strategies to a number of engineering disciplines.

The second consideration is finding metacognitive strategies for activating mathematical reasoning to overcome the influence of intuition; this is, of course, related to the first consideration. The nature of the interaction between intuition and mathematical reasoning is not fully understood (Chaiken & Trope, 1999; Segalowitz, 2007). However, recent research suggests that metacognitive difficulty activates analytic reasoning and overcomes intuitive errors (Alter et al., 2007). In the above mentioned research, difficulty and disfluency are introduced in an information processing phase in order to activate analytic reasoning. Neuroscientific evidence suggests that disfluency triggers the anterior cingulate cortex (Boksman et al., 2005), a cue that activates the prefrontal cortex responsible for deliberative and effortful thought (Botvinick, Braver, Carter, Barch, & Cohen, 2001; Lieberman, Gaunt, Gilbert, & Trope, 2002). Metacognitive strategies are widely applied in self-regulated learning (Winne & Perry, 2000). An agile teaching method is designed to help students utilize metacognitive strategies for activating mathematical reasoning in a variety of engineering problem-solving contexts (Arakawa, & Yukita, 2006; Chun, 2004).

In general, many strategies can be used in teaching math to engineering students, including problem-based learning (Barell, 2007; Duch, 2008; Kaminski, Sloutsky, & Heckler, 2008; Savin-Baden, 2003), technology-based learning (Trondsen, 1998), game-based learning (Prensky, 2004; Van, 2008), community-based learning (Owens & Wang, 2008), work-based learning (Bailey 2003; Cunningham, Dawes & Bennett, 2004), inquiry-based learning (Eick & Reed, 2002; Educational Broadcasting Corporation, 2008), project-

based learning (Helic, Maurer, & Scerbakov, 2004; The George Lucas Educational Foundation, 2008), team-based learning (Michaelsen, Knight & Fink, 2008), web-based Learning (Lee & Baylor 2006; O'Neil & Perez 2006), and participatory learning (Barab, Hay, Barnett, & Squire, 2001). There is no conflict between these strategies and agile teaching; an agile method can combine with any of the strategies for effective teaching.

A third consideration for this paper is to define the major issues involved and to set the stage for conducting experiments for measuring the effects of agile teaching on learning mathematical reasoning. An understanding of interactions between the two systems is essential for designing such experiments (Bodenhausen, Macrae, & Sherman, 1999).

Access to Mathematical Reasoning

Access to mathematical reasoning is usually achieved through education and training. The acquisition of skills in reasoning “selectively increases the accessibility of useful responses and of productive ways to organize information” (Kahneman, 2002). In the absence of such skills, there is no possibility of access to mathematical reasoning. Engineering students must acquire mathematical skills to demonstrate problem solving with access to analytic reasoning. Mathematical knowledge is highly structured; one needs to study algebra before calculus. Accessibility is a continuum and “some effortful operations demand more effort than others” (Kahneman, 2002). With this understanding, various courses of study in engineering, science, and technology are designed for adequate skill acquisition and subsequent practice in problem solving.

The pedagogical teaching of mental and mathematical skills to engineering students follows this model well. The beginning undergraduate frequently relies excessively on the intuition mode of thought. Through systematic, slow, deliberate, effortful teaching, judgmental skills are cultivated, options are evaluated and analytic capacity is developed. Students are amazed that focused work is required and that it does not come immediately. A variety of mathematical approaches have contributed to providing evidence for Kahneman’s proposition. Some examples are listed here with corresponding course numbers from the BS in Information Technology Management (ITM) program:

- Use of *gedankenexperiment* or thought experiments that Einstein made so famous, (Aspect, Grangier, & Roger, 1982) ITM470, ITM475
- Learning powers of ten notations. ITM320, ITM470, ITM475
- Learning dimensional analysis. ITM420, ITM470, ITM475
- Learning orders of magnitude estimation. ITM440, ITM470
- Witnessing the power and “mathematical soundness” of Abelian Group theory to relational database normalization. TM470, ITM475
- Virtual configurations. ITM320, ITM440, ITM470, ITM475

These have been applied to various courses in the BS in ITM program beginning with ITM320, Information Technology Management, and advancing through ITM475, Information Security Technologies. In a precourse quiz, students in ITM440, Database Principles, identified only a 27% level of knowledge of relational databases and no normalization capability. Following the completion of the course, 86% of the students felt they had developed the necessary skills to normalize a relational database. “Sound mathematics” in the form of Abelian Group operations produces consistently accurate results in SQL database operations. Furthermore, a union (recombination) of all SQL data subsets will return the original set of data.

The varied learning styles of students must also be recognized and accommodated to optimize the acquisition of mathematical skills in engineering courses. We recognize that there are variations in listings of learning styles starting with some well-known styles (Gardner, 1983). Continued effort and assessment are being made to evaluate the degree to which Kahneman’s proposition holds where skills are developed for quick access in reasoning mode.

Metacognitive Strategies

Cognition about cognition is metacognition. Metacognitive strategies are processes that one uses to monitor and control one’s cognitive activities for ensuring that a goal, such as correct problem solving, is achieved (Brown, 1987). These processes help to regulate and oversee cognitive functions. Recent research demonstrates that metacognitive strategies are effective in reducing errors in problem-solving tasks requiring analytic reasoning (Alter et al., 2007). This research demonstrated that a metacognitive strategy gives a cue that the task is difficult or that one’s intuitive response is likely to be wrong, thereby activating more analytic processing.

Following this research one can predict that students who learn to use metacognitive strategy will be able to overcome their intuitive mistakes by utilizing mathematical reasoning, provided that they have access to mathematical reasoning. Our teaching strategy therefore combines two related goals: (1) to increase students' access to mathematical reasoning, and (2) to enable students to use metacognitive strategies to their advantage. In our math classes students not only acquire math knowledge and skills but also learn how to use metacognitive strategies in problem solving. Since mathematical reasoning is effortful, analytic, and deliberate, metacognitive strategies are beneficial to the students.

Some general metacognitive strategies applicable to all students include self-observation, self-judgment, and self-reaction. With these strategies students learn how to observe their own cognitive processes, assess their own progress, and take corrective steps when needed. Under self-observation students may ask themselves questions such as "What have I learned in the preceding class? Can I apply De Morgan's laws of distribution?" Metacognitive strategies have potentials for significantly improving learning mathematical reasoning. These strategies are designed to overcome errors in the intuitive mode of reasoning.

The stage is set for collecting data on the effectiveness of these strategies. At this time, anecdotal evidence of student performance has been utilized for adjusting our teaching strategies to make further improvements. We have adopted the agile teaching methodology that allows us to combine multiple strategies in multimodel, multicultural learning environments (Dey et al., 2007).

We have gone well beyond anecdotal evidence in our use of Tablet Personal Computers in certain engineering classes. We received a two-year Technology for Teaching—Higher Education Grant from Hewlett-Packard Corporation in 2007. In a number of classes, we have integrated use of Tablet PCs in the hands of every student, with interactive exercises integrated into the flow of the class to help students acquire mathematical reasoning skills associated with complex information structures. In this approach, a mathematical concept is first introduced to the class. Students are challenged with a problem that involves mathematical reasoning to solve immediately in class. Each student is required to develop an answer on his or her Tablet PC and submit it through a wireless connection to the instructor. The instructor has the choice of receiving these submissions on an anonymous basis or with each student's submission identified by name. Anonymous submissions are useful to help students overcome fear of submitting a wrong answer. The instructor can choose certain

answers to discuss with the whole class, to illustrate common errors in logic, or to show a particularly clever approach to solving a problem.

This approach introduces a high degree of agility into the teaching process. If the students are taking longer than anticipated to come up with their answers, the instructor may conclude that students do not understand the concept very well and go over the reasoning process with the whole class. If certain students are having problems, the instructor may choose to work with them individually or put them with another student who understands the process and can help the individual having a problem. Use of Tablet PCs with appropriate software adds a great deal of agility to the teaching process.

How the Teaching Process Works

The following example illustrates how the teaching process works. One component of WCM 605—Information Privacy and Security in Wireless Systems teaches students how to generate “strong” passwords for user authentication (Yan, Blackwell, Anderson, & Grat, 2004). Students are also taught seven principles of generating strong passwords, as shown in Table 1. They are then taught a mechanism for generating strong passwords that involves complex mathematical reasoning. They start by thinking up a phrase that is relatively easy to remember and then extracting a password from that phrase by taking the first letter of some words and turning other words into numbers or special characters. For example, a password generation phrase might be “My three favorite months are March (3), June (6) and December (12).” The extracted password could be “M3fmrM3J6&D12.” This thirteen-character password is very difficult to guess or break. It complies with Rules 1–4. Because the phrase is easy to remember, it is easy for a user to comply with rules 6 and 7: “Don’t write it down” and “Don’t tell anyone.” And users are more willing to comply with Rule 5, “Change the password regularly,” when they can generate good passwords using this approach. Research has shown that passwords generated from mnemonic phrases are at least as strong as long random passwords that are computer-generated, but that is beyond the scope of this paper.

This mechanism for generating passwords was taught to WCM 605 classes in January, July, and October of 2007 by simply presenting the concept in class and leaving it up to students to experiment with it on their own. In the January 2008 and July 2008 WCM 605 classes, students were required to generate a passphrase on their Tablet PC in class, then extract a password from it and submit both the passphrase and the password to the instructor, as discussed above. To help those

who got it wrong, several of the anonymously submitted passphrases and passwords were discussed.

Table 1
Principles of Strong Passwords

1. Use characters other than just A-Z.
2. Choose long passwords.
3. Avoid names or words in any dictionary.
4. Choose an unlikely password.
5. Change the password regularly.
6. Don't write it down.
7. Don't tell anyone else.

We assessed the impact of these real-time, in-class exercises through midterm and final exam questions. One of the exam questions for all WCM 605 classes required students to generate a passphrase, extract a password from it, then discuss how it satisfied the requirements for strong passwords. Exam scores on this question improved from 22% correct answers for the October 2007 class to 88% correct in January 2008 and 95% correct for the July 2008 class.

The use of Tablet PCs with interactive software in class introduced a metacognitive strategy that forced students to use or apply concepts almost immediately after the concepts were taught. As a result, their skill in employing the new concepts was made much more accessible to them. We tested this hypothesis more broadly with a number of other questions dealing with concepts such as expressing a digital string as a polynomial; encrypting and decrypting a short message using substitutions and transpositions; using a complex structure known as a Vigenère tableau in encryption and decryption; and using cipher block chaining for encryption. Results from specific exam questions in the October 2007 class showed that these were all difficult skills for students to acquire. January and July 2008 results of the same questions (with details of the questions suitably altered to prevent cheating), showed dramatic improvement.

Table 2 shows that on the average, the number of students answering the questions correctly improved from an average of 18% correct answers on these five questions in October 2007 exams to a weighted average of 81% correct answers on the combined results of January 2008 and July 2008 exams, when the students were first

given real-time, in-class exercises to help them learn the concept to a sufficient depth to make the skill accessible. In addition, the overall weighted average of grades on the combined results of the January 2008 and July 2008 midterm exams improved by nearly 7.6% from 77.2% to 85.27%. These results are based on a combined enrollment in the two classes of 37 students.

Table 2
Improvement in Mathematical Reasoning

	% of Students Answering Correctly	% of Students Answering Correctly	% of Students Answering Correctly	Weighted Average Improvement
Description of Question	Oct-07	Jan-08	Jul-08	Change
Eselbrücke	22%	88%	95%	70%
Use Vigenère Tableau	11%	81%	89%	74%
Polynomial Representation	33%	75%	95%	53%
Encrypt Short Message	11%	69%	88%	68%
Cipher Block Chaining	11%	50%	40%	33%
Average	18%	73%	81%	60%
Number of Students	9	16	21	
Avg Grade overall	77.18%	84.10%	85.27%	7.58%

Table 2 uses October 2007 as a base with only 9 students. Ideally, we would like to have had a larger number of students in the base. However, these results are so encouraging that we have not been willing to penalize students by running a class without using the Tablet PCs, solely to increase the size of the base sample. Unfortunately data from a July 2007 WCM 605 class was not collected in sufficient detail to analyze individual questions. However, the average grade of the mid-term exam, taken by ten students, in July 2007, was 80.3%. The use of the Tablet PC approach to teaching the most difficult concepts was undertaken because of the recognition of

difficulties encountered by students in both the July and October classes in absorbing these concepts.

The data in Table 2 show some variation of results across the particular questions studied. For example, the ability to encrypt a simple message by hand improved from 69% correct in the January 2008 class to 88% in July, while the ability to write a binary number as a polynomial expression declined from 50% in the January 2008 class to 40% in the July 2008 class. Only 11% of the students answered these questions correctly in the base October 2007 class.

Teaching of simple encryption by substitution followed a similar pattern to that discussed above for password generation. Students were taught the basic building blocks of encryption: substitution and transposition. They were given an exercise in class to encrypt the text, "I ENJOY THE SAN DIEGO ZOO" with a substitution algorithm of the form $c_i = E(p_i) = p_i + n$, where p_i is the i^{th} letter of the plaintext (the text to be encrypted), and $E(p_i)$ is the encrypted value of the i^{th} letter of the ciphertext c_i . Students were instructed to use $n=5$ for the exercise. The correct result of the encryption is "N JSOTD YMJ XFS INJLT ETT."

It usually takes students no more than five minutes to do the encryption in class and submit it wirelessly to the instructor. Errors are easy to spot and common errors can be corrected quickly by the instructor. Students are also taught that 50% of all English text is one of the six letters A, E, I, N, O, or T and to use that information, along with common words like "the" and double letters like "oo" as a starting point for decrypting text that has been encrypted using a substitution algorithm. They are then given a decryption problem in their exam.

In the January 2008 class, the students were given an exam question that required them to decrypt a short message and find the value of the size of the shift - n . The specific problem and answer were:

The following ciphertext has been derived from a simple substitution cipher of the form $C_i = P_i + N$. Find the value of N that decrypts the ciphertext, decrypt it, and write the plaintext below. (The numbers and letters below the ciphertext are there to make your task easier).

YMJ BFYJW NS YMJ UTTQ NX AJWD HTTQ

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
1 2 3 4 5 6 7 8 9 10 1 2 3 4 5 6 7 8 9 20 1 2 3 4 5 6

Answer: "THE WATER IN THE POOL IS VERY COOL", $N = 5$

Of the January 2008 students 69% were successful in decrypting the message. The phrase to be decrypted is varied with each exam, to prevent students in one class from passing the answer to students in later classes. The plaintext result of the problem given to the July 2008 class was "**LOOPS IN LOOPS ARE COMMON IN CODE.**" As shown in Table 2, 88% of the students were successful in decrypting the ciphertext in July 2008.

To date we have taught the following four courses in the MSWC program using Tablet PCs: WCM 601—Digital Wireless Fundamentals, WCM 604—Wireless Coding and Modulation, WCM 605—Wireless Systems Security, and WCM 610—Next Generation Wireless Systems. A newly developed course, WCM 612—Wireless Economics Topics, is currently being taught with Tablet PCs, using similar techniques.

Following receipt of approval of the instrument by the National University Institutional Review Board, students in the April 2008 WCM 604 course and the July 2008 WCM 605 course were invited to complete surveys about their use of Tablet PCs in class. Results of the nine questions will be discussed more fully when we have collected data from more classes, but two survey questions are particularly relevant to this paper. Students were asked to score their agreement/disagreement with the following two statements on a five-point Likert scale:

A. "Classes taught with a Tablet PC keep me more engaged in learning than classes taught with desktop or laptop computers for students. "

and

B. "Use of Tablet PCs by students enabled me to learn new concepts better/faster because I was able to understand the way other students reasoned about a problem."

The average score from the April 2008 WCM 601 class was 4.4 for statement A and 4.2 for statement B. The average score from the July 2008 WCM 605 class was 4.18 for A and 4.09 for B. We believe this supports our contention that this teaching technique makes material more accessible to the reasoning needed to learn complex mathematical concepts. We look forward to collecting the same data from more courses to better support this contention.

Concluding Remarks: Setting the Stage for Experimental Studies

As we learn more about learning, we understand its scientific aspects based on the recent contributions from neuroscience, psychology and cognitive science (Bransford, Brown, & Cocking, 1999). The emerging notion of interactions between intuition and mathematical reasoning is important for teaching environments. It is possible that in certain problem-solving approaches, people use random guessing; that is, they use neither intuition nor mathematical reasoning. Thus, questions can be raised about the validity of the classical dual process theory for unrestricted problem-solving circumstances. However, the focus of this paper has been narrow in the sense that it has tried to find strategies for avoiding mistakes of intuitive mode without addressing mistakes of other possible modes of cognitive function. Teaching strategies have been suggested for increasing students' access to controlled mathematical reasoning. Teachers need to perform their teaching with sufficient agility in order to adjust their strategies to learner's goals, styles and preferences.

With deeper understanding of the issues, we are now better prepared for conducting our experimental studies on the effectiveness of our agile teaching methodology. A special strategy we will be investigating will introduce the use of games in teaching certain engineering subjects through a project titled, Virtual Apprenticeship Through Mobile Gaming: Facilitating STEM Learning Through Game Design. One of our major goals in this work is to change students' focus from learning theory to learning practical application of theory through simulation games—i.e. to acquire the skills to apply the theory. We will expose students to real-world challenges that they will soon face in their careers by extending their learning through the introduction of simulation games in virtual environments. Through simulation gaming, we will provide an environment of problem-based learning that promotes constructive competition among students. These games will simulate real-world organizational dynamics and improve retention of complex concepts. This process will involve mapping fundamental theories of engineering to rules and procedures expressed through game play. Effectively, the students will design and build the games and then play them. We intend to use this approach, for example, to teach wireless communications network design and to introduce competition among groups of students, working together to design the "best" network.

Acknowledgments

The authors are grateful to Alireza Farahani, and Hassan Badkoobehi for their participation in the discussion of issues raised in this paper and their suggestions and comments on earlier versions of the paper. The authors are also thankful to anonymous reviewers of the Journal of Research in Innovative Teaching for valuable suggestions and comments. The project Virtual Apprenticeship Through Mobile Gaming: Facilitating STEM Learning Through Game Design mentioned in this article is made possible through a Technology for Teaching Leadership Grant received from the Hewlett-Packard Corporation in June 2008.

References

- Alter, A. L., Openheimer, D. M., Epley, N., & Eyre, R. (2007). Overcoming Intuition: Metacognitive difficulty activates analytic reasoning. *Journal of Experimental Psychology*, 136(4), 569–576.
- Arakawa, S., & Yukita, S. (2006). An effective agile teaching environment for java programming courses. *Proceedings of the 36th Annual Frontiers in Education Conference*, pp. 13–18.
- Aspect, A., Grangier, P., & Roger, G. (1982). Experimental realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A new violation of Bell's Inequalities. *Physical Review Letters*, Vol. 49, No. 2.
- Bailey, T. R. (2003). *Working knowledge: Work-based learning and education reform*. New York: RoutledgeFalmer.
- Barab, S. A., Hay, K. E., Barnett, M., & Squire, K. (2001). Constructing virtual worlds: Tracing the historical development of learner practices, *Cognition and Instruction*, 19(1), 47–94
- Barell, J. (2007). *Problem-based learning: An inquiry approach* (2nd Edition). Thousand Oaks: Corwin Press.
- Bless, H., & Schwarz, N. (1999). Sufficient and necessary conditions in dual process models: The case of mood and information processing. In S. Chaiken & Y. Trope (Eds.), *Dual-process theories in social psychology* (pp. 423–440). New York: Guilford Press.
- Bodenhausen, G. V., Macrae, C. N., & Sherman, J. W. (1999). On the dialectics of discrimination: Dual processes in social stereotyping. In S. Chaiken & Y. Trope (Eds.), *Dual-process theories in social psychology* (pp. 271–290). New York: Guilford Press.
- Boksman, K., Theberge, J., Williamson, P., Drost, D. J., Malla, A., Densmore, M., et al. (2005). A 4.0-T fMRI study of brain connectivity during word fluency in first-episode schizophrenia. *Schizophrenia Research*, 75, 247–263.
- Borman, K. M. (2005). *Meaningful urban education reform : confronting the learning crisis in mathematics and science*. Buffalo: State University of New York

Press.

- Botvinick, M. M., Braver, T. S., Carter, C. S., Barch, D. M., & Cohen, J. D. (2001). Conflict monitoring and cognitive control. *Psychological Review*, 108, 624–652.
- Bransford, J. D., Brown, A. L., & Cocking, R. R. (1999) *How people learn: Brain, mind, experience, and school*. Washington, DC: National Academy Press.
- Brown, A. L. (1987). Metacognition, executive control, self-regulation, and other more mysterious mechanisms. In F. E. Weinert & R. H. Kluwe (Eds.), *Metacognition, motivation, and understanding* (pp. 65–116). Hillsdale, NJ: Erlbaum.
- Chaiken, S. (1980). Heuristic versus systematic information processing and the use of source versus message cues in persuasion. *Journal of Personality and Social Psychology*, 39, 752–766.
- Chaiken, S. & Trope, Y. (Eds.). (1999). *Dual-process theories in social psychology*. New York: Guilford Press.
- Chun, A. (2004). The agile teaching/learning methodology and its e-learning platform. In *Lecture Notes in Computer Science: Advances in Web-Based Learning*, Vol. 3143 (pp.11–18). Heidelberg: Springer-Verlag.
- Cunningham, I., Dawes, G., & Bennett, B. (2004). *The handbook of work-based learning*. Adershot: Gower Publishing.
- Dey, P. P., Amin, M., Bright, M., Datta, A., Akhtar, S., Cruz, A., Wyne, M. F., Kennedy, P., Olson, P., Snyder, R., Farahani, A., Afuwape, S., & Badkoobehi, H. (2008). Multi-model multi-strategy teaching/learning in science, engineering and technology. *Proceedings of the International Computer Science and Technology Conference (ICSTC-2008)*, San Diego.
- Duch, B. (2008). Problem-based learning. Retrieved February 10, 2008 from <http://www.udel.edu/pbl/>.
- Educational Broadcasting Corporation.. (2008). Inquiry-based learning, Retrieved February 10, 2008 from <https://www.thirteen.org/edonline/concept2class/inquiry/inde.html>
- Eick, C. J., & Reed, C. J. (2002). What makes an inquiry oriented science teacher? The influence of learning histories on student teacher role identity and practice. *Science Teacher Education*, 86, 401–416.
- Evans, J. S. B. T. (2003). In two minds: Dual-process accounts of reasoning. *Trends in Cognitive Sciences*, 7, 454–459.
- Gardner, H. (1983). *Frames of mind: The theory of multiple intelligences*. New York: Basic Books.
- George Lucas Educational Foundation, The. (2008). Project based learning, Retrieved February 10, 2008 from <http://www.edutopia.org/projectbasedlearning>

- Helic, D., Maurer, H., & Scerbakov, N. (2004). Knowledge transfer processes in a modern WBT system. *Journal of Network and Computer Applications*, 27(3), 163–190.
- Higgins, E. T. (1996). Knowledge activation: Accessibility, applicability, and salience. In E. T. Higgins & A. Kruglanski (Eds.), *Social psychology: Handbook of basic principles* (pp. 133–168). New York: Guilford Press.
- Kahneman, D. (2002). Maps of bounded rationality: A perspective on intuitive judgment and choice (Nobel Prize Lecture). In Tore Frängsmyr (Ed.). *Les Prix Nobel: The Nobel Prizes 2002*, pp. 416-499.
- Kahneman, D., & Frederick, S. (2002). Representativeness revisited: Attribute substitution in intuitive judgment. In T. Gilovich, D. Griffin, & D. Kahneman (Eds.), *Heuristics and biases: The psychology of intuitive judgment* (pp. 49–81). New York: Cambridge University Press.
- Kaminski, J. A., Sloutsky, V. M., & Heckler, A. F. (2008, 25 April) Learning theory: The advantage of abstract examples in learning math. *Science*, Vol. 320.
- Kriegler, S. (2008). Just what is algebraic thinking? Retrieved September 10, 2008 from <http://www.math.ucla.edu/~kriegler/pub/algebrat.html>.
- Lee, M. & Baylor, A. L. (2006) Designing Metacognitive Maps for Web-Based Learning. *Educational Technology & Society*, 9 (1), 344-348.
- Lieberman, M. D., Gaunt, R., Gilbert, D. T., & Trope, Y. (2002). Reflection and reflexion: A social cognitive neuroscience approach to attributional inference. In M. P. Zanna (Ed.), *Advances in experimental social psychology* (Vol. 34, pp. 199–249). New York: Academic Press.
- Michaelsen, L. K., Knight, A. B., & Fink, D. L. (2008). *Team-based learning: A transformative use of small groups in college teaching*. Sterling, VA: Stylus Publishing.
- Mingus, T. T. Y., & Grassl, R. M. (1998). Algorithmic and recursive thinking: Current beliefs and their implications for the future. In *The teaching and learning of algorithms in school mathematics* (pp. 32–43). Reston, VA: NCTM.
- O'Neil, H. F., & Perez, R. S. (2006). *Web-based Learning: Theory, research, and practice*. Hillsdale, NJ: Erlbaum.
- Owens, T. R., & Wang, C. (2008). Community-based learning, Retrieved February 10, 2008 from <http://www.nwrel.org/scpd/sirs/10/t1008.html>
- Petty, R. E., & Cacioppo, J. T. (1986). The elaboration likelihood model of persuasion. In L. Berkowitz (Ed.), *Advances in experimental social psychology* (Vol. 19, pp.123–205). New York: Academic Press.
- Prensky, M. (2004). *Digital game-based learning*. New York: McGraw-Hill.

- Savin-Baden, M. (2003). Facilitating problem-based learning: *Illuminating perspectives*. Buckingham: SRHE/Open University Press.
- Segalowitz, S. (2007). *Knowing before we know: Conscious versus preconscious top-down processing and a neuroscience of intuition*. *Brain and Cognition*, 65(2), 143–144.
- Tetlock, P. E., & Lerner, J. S. (1999). The social contingency model: Identifying empirical and normative boundary conditions for the error and bias portrait of human nature. In S. Chaiken & Y. Trope (Eds.), *Dual-process theories in social psychology* (pp. 571–585). New York: Guilford Press.
- Trondsen, E. (1998). Technology-based learning, Retrieved February 10, 2008 from <http://www.sric-bi.com/LoD/summaries/LoDRpt01.shtml>
- Van, R. E. (2008). Game-based learning. *EDUCAUSE Review*, 41(2), 16–30.
- Wallis, C. (2008, February 25). How to make great teachers? *Time*, 171 (8).
- Winne, P. H., & Perry, N. E. (2000). Measuring self-regulated learning. In P. Pintrich, M. Boekaerts, & M. Seidner (Eds.), *Handbook of self-regulation* (pp. 531–566). Orlando, FL: Academic Press.
- Yan, Y., Blackwell, A., Anderson R. & Grant, A. (2004) Password memorability and security: Empirical results. *IEEE Security & Privacy*, 4(5), 25–31.

About the Authors

Pradip Peter Dey
PhD, Professor
Department of Computer Science and Information Systems
National University
San Diego, California
pdey@nu.edu
Major research interests: Design and implementation of intelligent systems, software engineering, mathematical reasoning

Ronald P. Uhlig
PhD, Associate Professor
Chair, Department of Computer Science and Information Systems
National University
San Diego, California
ruhlig@nu.edu
Major research interests: Wireless communications, telecommunications, data networks, intelligent networks, mobile applications, engineering education, educational technology

Mohammad N. Amin
PhD, Professor
Department of Applied Engineering
National University
San Diego, California
mamin@nu.edu

Major research interests: Mathematical reasoning, agile teaching, database, sensors and wireless applications

Arun Datta
PhD, Senior Fellow
Community Research Institute (NUCRI), Adjunct Professor
National University
La Jolla, California
adatta@nu.edu

Major research interests: Bioinformatics and health Informatics, grid technology and its application in biological domain, educational tools and technologies

Gordon W. Romney
PhD, Professor
Department of Computer Science and Information Systems
National University
San Diego, California
gromney@nu.edu

Major research interests: Information assurance and cybersecurity, information integrity and confidentiality, 3D graphics, anonymization of data, remote wireless sensors, virtualization, Ruby on Rails, and JRuby

Thomas M. Gatton
PhD, Professor
Department of Applied Engineering
National University
San Diego, California
tgatton@nu.edu

Major research interests: Design and construction automation, artificial intelligence, software engineering, intelligent tutorial systems

Mudasser F. Wyne
PhD, Professor
Department of Computer Science and Information Systems
National University
San Diego, California
mwyne@nu.edu

Major research interests: Mathematical reasoning, database management system, information systems, mobile database

Albert P. Cruz
M.B.A., M.A., Instructor
Department of Computer Science and Information Systems
National University
San Diego, California
acruz@nu.edu

Major research interests: Knowledge management, decision making